FRIDAY: 1 December 2017

Time Allowed: 3 hours.

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show ALL your workings.

**QUESTION ONE**

(a) Outline four applications of mathematical functions in business. 

(b) Highlight the four components of a time series.

(c) A survey was conducted on 800 households to determine their preference for three consumer goods, namely Fex, Gex and Mex. The results of the survey were as follows:

- 230 households preferred Fex.
- 245 households preferred Gex.
- 325 households preferred Mex.
- 30 households preferred all the three goods.
- 70 households preferred Fex and Mex.
- 110 households preferred Fex only.
- 185 households preferred Mex only.

**Required:**

(i) Present the above information in aven diagram. 

(ii) The number of households that preferred Fex and Gex.

(iii) The probability that a household selected at random does not prefer any of the three goods.

(d) Sosta Baridi Limited has a computerised customer billing system. Customers’ accounts are classified as being paid, delinquent or bad debt. The company has a total of 1,500,000 customer accounts. A computer program was developed to display transition of accounts among the three categories. The output from the program is summarised below:

<table>
<thead>
<tr>
<th></th>
<th>Paid</th>
<th>Delinquent</th>
<th>Bad debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Delinquent</td>
<td>285,000</td>
<td>15,000</td>
<td>0</td>
</tr>
<tr>
<td>From Bad debt</td>
<td>20,000</td>
<td>700,000</td>
<td>30,000</td>
</tr>
<tr>
<td>From Paid</td>
<td>0</td>
<td>0</td>
<td>450,000</td>
</tr>
</tbody>
</table>

The above transitions took place between 31 December 2015 and 31 December 2016.

**Required:**

The percentage of customers that will be in each of the three categories of accounts as at 31 December 2017. (6 marks)

(Total: 20 marks)
(a) Enumerate three circumstances under which the Poisson distribution is most applicable. (3 marks)

(b) A company produces two types of electric components whose information has been provided to you as follows:

<table>
<thead>
<tr>
<th>Component X</th>
<th>Component Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean life in hours</td>
<td>1,600</td>
</tr>
<tr>
<td>Standard deviation in hours</td>
<td>132</td>
</tr>
<tr>
<td>Sample size</td>
<td>120</td>
</tr>
</tbody>
</table>

Required:
Determine whether the quality of the two types of electric components differ significantly. (Use a significance level of 95%). (3 marks)

(c) A survey conducted on citizens of a certain country to determine the annual per capita income indicated that the annual income of the citizens is normally distributed with a mean of Sh.980,000 and a standard deviation of Sh.160,000. One citizen was randomly selected from the country.

Required:
The probability that the annual income of the citizen:

(i) Is greater than Sh.500,000. (2 marks)

(ii) Is greater than Sh.1,220,000. (2 marks)

(iii) Lies between Sh.852,000 and Sh.1,100,000. (2 marks)

(d) Excellent Products Limited manufactures four products, A, B, C and D, using four machines, M1, M2, M3, and M4. The total outputs of the four products from the four machines are as shown below:

<table>
<thead>
<tr>
<th>Machines</th>
<th>Output (units “000”)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>M1</td>
<td>12</td>
</tr>
<tr>
<td>M2</td>
<td>18</td>
</tr>
<tr>
<td>M3</td>
<td>16</td>
</tr>
<tr>
<td>M4</td>
<td>14</td>
</tr>
</tbody>
</table>

The company intends to assign the production of each output to a particular machine.

Required:
Advise the management of Excellent Products Limited on the best assignment that will maximise production. (4 marks)

(e) The demand and total cost functions (in thousands of shillings) of a certain company that deals in the manufacture of a product named “Exe” are given as follows:

\[
P = 75 - 0.18Q \\
TC = 80Q + 5Q^2 - 0.03Q^3
\]

Where:
P is the unit selling price.
Q is the quantity demanded in units.
TC is the total cost.

Required:
(i) The profit function. (1 mark).

(ii) The output level that would maximise profit. (3 marks)

(Total: 20 marks)

QUESTION THREE

(a) Distinguish between “regression analysis” and “correlation analysis”. (2 marks)

(b) Summarise two applications of rank correlation. (2 marks)
\[ y = ab^x \]

Where:  
- \( y \) is the advertising cost in thousands of shillings
- \( a, b \) are constants over a period of seven years.
- \( x \) is the period under consideration.

The actual advertising cost of the enterprise over a seven-year period is given as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>32</td>
<td>47</td>
<td>65</td>
<td>92</td>
<td>132</td>
<td>190</td>
<td>275</td>
</tr>
<tr>
<td>(Sh. “000”)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Required:
(i) The advertising cost function of the enterprise, using the normal equation.
(ii) The advertising cost of the enterprise in year 8.

(d) The management of New Era Computer Systems Limited is planning to launch a new product branded Zimsang. The fixed cost of Zimsang is Sh.80,000. However, the selling price, variable costs, and annual sales volume of Zimsang are uncertain. The data below relate to product Zimsang:

<table>
<thead>
<tr>
<th>Unit selling price (Sh.)</th>
<th>Probability</th>
<th>Variable cost (Sh.)</th>
<th>Probability</th>
<th>Sales volume (units)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.25</td>
<td>20</td>
<td>0.25</td>
<td>40,000</td>
<td>0.30</td>
</tr>
<tr>
<td>80</td>
<td>0.45</td>
<td>40</td>
<td>0.55</td>
<td>60,000</td>
<td>0.35</td>
</tr>
<tr>
<td>100</td>
<td>0.30</td>
<td>60</td>
<td>0.20</td>
<td>100,000</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Required:
Simulate the average profit of product Zimsang on the basis of 10 trials. Use the following random numbers:

81 32 60 94 46 31 67 25 24 10 40 02 39
68 08 59 66 90 12 64 79 31 86 68 82 89
25 11 98 16

QUESTION FOUR
(a) In a certain busy business facility, the mean arrival rate of clients is 800 clients per hour. The mean service rate is 820 clients per hour. The facility operates between 6:00 a.m. and 6:00 p.m. every day. The management of the facility are concerned about the average number of customers in the queueing system and wish to improve the facility in order to serve an average of 847 clients per hour. The cost of improving the facility amounts to Sh.18,500 per day. Each hour lost costs the facility Sh.125.

Required:
(i) The average waiting cost per day.
(ii) Advise the management on whether they should improve the facility.
(iii) Compare the probabilities that the total number of clients in the queue and those being served is greater than 17 in the existing and in the improved facilities.

(b) Two airlines, K and Q are interested in increasing their market shares. Airline K has three available strategies, advertising its special fare, advertising its unique features, or advertising its safety record. On the other hand, Airline Q also has three available strategies: do nothing, advertise its special fare, or advertise its special features.

The matrix below shows the gains and losses associated with the different available strategies in millions of shillings. Positive values favour Airline K and negative values favour Airline Q.

<table>
<thead>
<tr>
<th>Airline Q</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>-30</td>
<td>0</td>
<td>-90</td>
</tr>
<tr>
<td>K2</td>
<td>-40</td>
<td>15</td>
<td>-20</td>
</tr>
<tr>
<td>K3</td>
<td>80</td>
<td>20</td>
<td>-50</td>
</tr>
</tbody>
</table>
K₁ = Advertise special fare.
K₂ = Advertise unique features.
K₃ = Advertise safety record.
Q₁ = Do nothing.
Q₂ = Advertise special fare.
Q₃ = Advertise special features.

Required:
(i) The optimal strategies for each airline. (7 marks)
(ii) The value of the game. (1 mark)

(Total: 20 marks)

QUESTION FIVE
(a) Explain the following terms as used in game theory:
(i) Saddle point. (1 mark)
(ii) Dominance. (1 mark)

(b) Suggest two areas in which advanced information technology could be used to solve quantitative analysis problems. (2 marks)

(c) The data below relate to normal duration and cost along with crash duration and cost for each activity of a certain project.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Normal duration (Days)</th>
<th>Crash duration (Days)</th>
<th>Normal cost (Sh.)</th>
<th>Crash cost (Sh.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2</td>
<td>6</td>
<td>4</td>
<td>2,800,000</td>
<td>3,800,000</td>
</tr>
<tr>
<td>1 – 3</td>
<td>8</td>
<td>5</td>
<td>4,000,000</td>
<td>5,600,000</td>
</tr>
<tr>
<td>2 – 3</td>
<td>4</td>
<td>2</td>
<td>2,200,000</td>
<td>3,000,000</td>
</tr>
<tr>
<td>2 – 4</td>
<td>3</td>
<td>2</td>
<td>1,600,000</td>
<td>2,800,000</td>
</tr>
<tr>
<td>3 – 4</td>
<td>Dummy</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3 – 5</td>
<td>6</td>
<td>3</td>
<td>1,800,000</td>
<td>3,200,000</td>
</tr>
<tr>
<td>4 – 6</td>
<td>10</td>
<td>6</td>
<td>5,000,000</td>
<td>7,000,000</td>
</tr>
<tr>
<td>5 – 6</td>
<td>3</td>
<td>2</td>
<td>1,000,000</td>
<td>1,600,000</td>
</tr>
</tbody>
</table>

The indirect cost of the project is Sh.600,000 per day.

Required:
(i) The normal duration and the corresponding total cost. (6 marks)
(ii) The minimum duration and the corresponding total cost. (8 marks)
(iii) The optimum duration and the corresponding total cost. (2 marks)

(Total: 20 marks)